MATH95 7.1 Exponents

https://www.youtube.com/watch?v=1_DmWqEpSmM&list=PL9dj44OpeMZEgN4Kqgg7mF0AJOEn2Fzb8&index=19

Recall: Exponents are used to represent repeated multiplication.

Example 1: Simplify the following.

a) \((-3)^2\)

b) \(-3^2\)

Remember that when you multiply an ______________ number of negatives together, you get a ______________, and when you multiply an ______________ number of negatives together, you get a _____________________.

Review:

- Dividing is like multiplying by the ________________.
- Exponents are used to represent repeated _________________.

Note: \(x \neq 0\)

\[x^4 = \]
\[x^3 = \]
\[x^2 = \]
\[x^1 = \]
\[x^0 = \]

Then, a negative exponent is used to represent repeated division (in other words, repeated multiplication by the reciprocal).

Consider \(2^5\):

Now, consider \(2^{-5}\).
The “-” on the exponent ONLY means to take the reciprocal of the base. It does affect the sign of the base!!!

Example 2: Simplify the following, using positive exponents only.

a) \( y^{-9} \)  

b) \( xy^{-9} \)

c) \( 5^{-2} \)  

d) \( \left( \frac{1}{3} \right)^{-2} \)

e) \( (-3)^{-3} \)  

f) \( (2x)^{-3} \)

Consider \( \frac{1}{x^{-2}} \).

So… \( a^{-n} = \frac{1}{a^n}, \quad a \neq 0 \)

When your expression involves a fraction, a negative exponent in the numerator means you can move that factor to the denominator (and the exponent is then positive) and vice versa.

Consider \( x^4 \cdot x^2 \).

Let \( m, n \) be positive integers.

**Product Rule:** \( a^m \cdot a^n = a^{m+n} \)

Note these are factors, since they are being multiplied. There is no “exponent rule” for terms being added or subtracted.
Example 3: Simplify:

a) \(2^2 \cdot 2^4\)

b) \((-4x^3y)(2x^2y)\)

Consider \((y^2)^3\).

| Power Rule: \((a^m)^n = a^{mn}\) |

Example 4: Simplify: \((x^4)^4\)

Consider \((xy^2)^3\).

| Power of a Product: \((ab)^m = a^m b^m\) |

Note these are factors, since they are being multiplied. There is no “exponent rule” for terms being added or subtracted.

Example 5: Simplify: \(2(4xy^6)^2\)

Consider \(\frac{x^4}{x^2}\).

| Quotient Rule: \(\frac{a^m}{a^n} = a^{m-n}, a \neq 0\) |

Example 6: Simplify: \(\frac{2x^2y^5}{xy^4}\)
Consider \(\left(\frac{x}{y}\right)^4\).

**Power of a Quotient:** 
\[
\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, \quad b \neq 0
\]

**Example 7:** Simplify: 
\[
\left(\frac{-2x^2}{y}\right)^4
\]

Consider \(\frac{x^3}{x^4}\).

**Zero Exponent:** 
\[a^0 = 1, \quad a \neq 0\]

Any real number (except for 0) raised to the “0” power is 1!

**Example 8:** Simplify:

a) \(\left(4xy^2\right)^0\) \quad b) \(5x^0 + 4y^0\)

**Example 9:** Simplify the following, using positive exponents only.

a) \(\frac{2x^{-2}}{y}\) \quad b) \(\frac{(a^{-2}b^2)^{-3}}{ab^{-2}}\)
Consider \( \frac{x^4 y^5}{x^3 y} \).

**Example 10:** Simplify the following, using positive exponents only.

a) \( \frac{x^2 y^9 z^4}{x^5 y^7} \)

b) \( \left( \frac{x^{-2} y}{2 y^{-3} x} \right)^{-2} \)

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**Summary of Exponent Rules**

If \( m \) and \( n \) are positive integers and \( a \) and \( b \) are real numbers for which the expressions exist:

- **Product Rule:** \( a^m \cdot a^n = a^{m+n} \)
- **Power Rule:** \( (a^m)^n = a^{mn} \)
- **Power of a Product:** \( (ab)^n = a^m b^m \)
- **Power of a Quotient:** \( \left( \frac{a}{b} \right)^m = \frac{a^m}{b^m}, b \neq 0 \)
- **Quotient Rule:** \( \frac{a^m}{a^n} = a^{m-n}, a \neq 0 \)
- **Zero Exponent:** \( a^0 = 1, a \neq 0 \)
- **Negative Exponent:** \( a^{-n} = \frac{1}{a^n}, a \neq 0 \)
**Review of Scientific Notation**

There are some values, especially those used in science and math fields, that are very large or small. Consider the value 1 billion = 1,000,000,000. This is a large number that doesn’t take too long to write out. But consider the value 1 googol. This is the number $10^{100}$ or 1 followed by 100 zeroes:

$$1 \text{ googol} = 10^{100} = 10,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000.$$

This would take awhile to write out if being used in an equation or application. To write very large or small numbers, we use **scientific notation**.

A positive number is written in scientific notation as:

$$a \times 10^r$$

where $1 \leq a < 10$ and $r$ is an integer power of 10

- If $r > 0$, the number is greater than or equal to 1 (move decimal of $a$ to the right $r$ places).
- If $r < 0$, the number is less than 1 (move decimal of $a$ to the left $r$ places).

Thus, $1 \text{ googol} = 10,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000$ in **standard form**

and $1 \text{ googol} = 1.00 \times 10^{100}$ in **scientific notation**

**Example 12:** Convert the following numbers to scientific notation.

a) 9,300,000,000,000 

b) 0.0000185

**Example 13:** Convert the following numbers to standard form.

a) $9.056 \times 10^{-4}$ 

b) $9.056 \times 10^{6}$

**Example 14:** Perform each indicated operation. Write each answer in standard form.

a) \[
\left(2.5 \times 10^{6}\right)\left(2 \times 10^{-6}\right)\]

b) \[
\frac{0.00025}{50,000}
\]

Note: Your homework for scientific notation will come from Appendix A1 in Hawkes. Do the A1 homework along with Section 7.1 homework!