In Chapter 4, we learned how to solve quadratic inequalities using a graph. In this section, we’ll learn how to solve nonlinear inequalities, specifically polynomial (which includes quadratic) and rational inequalities.

Example 1: Solve $x^4 \leq 9x^2$.

1. Write in standard form (zero on one side).

2. Find the values for which $f(x) = 0$ (the zeros of the function) and where $f(x)$ is undefined. All the values are called the critical values of the function because they represent the values at which the function potentially changes sign.

3. Use these critical values to break the number line into regions.

4. Choose a test point from each region and determine the sign of $f(x)$ in the region.

5. The solution set includes the regions that satisfy the inequality. If the inequality is $\leq$ or $\geq$, the solutions of the related equation are also part of the solution set. If the sign is $>$ or $<$, they are NOT included.
Example 2: Solve \( \frac{x + 2}{x - 4} \geq 1 \)

When the inequality is in standard form and there is a fraction, you must FIRST determine where the denominator equals 0. This \( x \) value would make the function undefined and is used to separate the number line into regions. It must also be excluded as a solution in the end!

Application

Example 3: The average daily cost of manufacturing \( x \) bicycles is given by \( C(x) = \frac{80x + 6000}{x} \). How many bicycles must be produced each day for the average cost to be no more than $100?