The perimeter of a square, \( p \), is always 4 times the length of a side, \( s \). In other words, \( p = 4s \). We can say that \( p \) varies \underline{\text{directly}} as (or with) \( s \). In other words, \( p \) is \underline{\text{directly proportional}} to \( s \).

**Direct Variation**

If there exists a nonzero constant, \( k \), such that \( y = kx \), then \( y \) \underline{varies \text{ directly proportional}} with \( x \) or \( y \) is \underline{directly proportional} to \( x \).

- \( k \) is called the \underline{constant of variation} or \underline{constant of proportionality}.

**Example 1:** If \( v \) varies directly with \( t \), find the constant of variation, \( k \), and then the direction variation equation.

\[ v = 16 \text{ when } t = 2 \]

There are other types of variation where \( k \) is called the \underline{constant of variation} or \underline{constant of proportionality}.

**Inverse Variation**

If there exists a nonzero constant, \( k \), such that \( y = \frac{k}{x} \), then \( y \) \underline{varies \text{ inversely proportional}} with \( x \) or \( y \) is \underline{inversely proportional} to \( x \).

**Example 2:** If \( y \) varies inversely with \( \sqrt{x} \), find the constant of variation, \( k \), and then the inverse variation equation.

\[ y = 4 \text{ when } x = 9 \]
Joint Variation

If there exists a nonzero constant, \( k \), such that \( y = kxz \), then \( y \) varies \_______________\ with \( x \) and \( z \) or \( y \) is \textbf{jointly proportional} to \( x \) and \( z \). (The product may consist of more variables than just \( x \) and \( z \).)

\textit{Example 3:} Find the constant of variation, \( k \), and the joint variation equation if \( T \) varies jointly with \( x \) and the square of \( d \).

\[
T = 18 \quad \text{when} \quad x = 1 \quad \text{and} \quad d = 3
\]

Combined Variation

Variation equations may also consist of some combination of joint, direct, and inverse variation.

\textit{Example 4:} The maximum weight that a rectangular beam can support varies jointly as its width and the square of its height and inversely as its length. If a beam 1/3 foot wide, 1 foot high, and 10 feet long can support 3 tons, find how much weight a similar beam can support if it is 1 foot wide, 1/3 foot high, and 9 feet long.