When solving equations, you will often need to work with radicals. We learn how in this section! The word “radical” comes from the Latin word *radix* which means __________.

**Question:** What numbers squared will give you 49?

**Square Roots**

**Square root of a:** number(s) that was squared to get *a*

- 7 is the ________________, or non-negative, square root of 49 and is denoted by:
- −7 is the negative square root of 49 and is denoted by:

**Question:** What is √49?

You cannot take the square root of a negative number in the real number system because there is no real number squared that will give a negative number!

1, 4, \( \frac{4}{9} \), 9, 81 are examples of ________________ because their square roots are rational numbers. Numbers such as √2 are irrational and we can only approximate the value of it using a calculator.

\[
\sqrt{2} \approx \]

**Cube Roots**

**Cube root of a:** number that was cubed to get *a*

For example,

\[2^3 = 8\] so….

\[(-2)^3 = -8\] so….

Notice that, unlike square roots, you can take the cube root of a negative number. The cube root of a negative number is negative. Also, if the radicand is not a perfect cube, you could use a calculator to approximate the value!
Example 1: Find the following.

a) \( \sqrt{121} \)  

b) \( -\frac{16}{\sqrt{25}} \)

c) \( \sqrt[3]{-125} \)  

d) \( \frac{\sqrt[3]{8}}{\sqrt{27}} \)

**nth Roots**

\[ n\sqrt{a} = b \quad \text{if and only if} \quad b^n = a \quad \text{where} \quad n\sqrt{a} \quad \text{is defined} \quad (n > 1, \quad n \text{ is an integer}) \]

**Even Index \((n)\)**

*Examples:*

Just like square roots where \( n = 2 \), the radicand with an even index must be non-negative or the result is not a real number.

Then, \( n\sqrt{a} \) (when \( n \) is even) is a ____________________________ real number.

**Odd Index \((n)\)**

*Examples:*

Just like cube roots where \( n = 3 \), the radicand with an odd index can be any real number.

Then, \( n\sqrt{a} \) (when \( n \) is odd) can be any real number.

Example 2: Find the following.

a) \( \sqrt[4]{16} \)  

b) \( \sqrt[3]{-32} \)
Consider $\sqrt{x^2}$. $x$ could be negative or positive (or zero!).
For example… $\sqrt{4^2} = \sqrt{(-4)^2} =$

Consider $\sqrt[3]{x^3}$. $x$ could be negative or positive (or zero!).
For example… $\sqrt[3]{2^3} = \sqrt[3]{(-2)^3} =$

Then, $\sqrt[n]{a^n} =$

Example 3: Find the following.

a) $\sqrt{a^2}$

b) $\sqrt[3]{a^3}$

There are some radical rules we can use to simplify expressions. Assume that all variables represent non-negative real numbers!

For your reference…

Perfect Powers

Perfect Squares 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169

Perfect Cubes 1, 8, 27, 64, 125, 216

Perfect Fourth Powers 1, 16, 81, 256, 625
Consider the radical expression: \( \sqrt{4} \cdot \sqrt{25} \)

Now consider the radical expression: \( \frac{\sqrt{4}}{\sqrt{25}} \)

If \( \sqrt{a} \) and \( \sqrt{b} \) are real numbers,

Product Rule for Radicals \( \sqrt{a} \cdot \sqrt{b} = \sqrt{ab} \)

Quotient Rule for Radicals \( \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \) if \( \sqrt{b} \) is not zero

A radical expression is in **simplest radical form** if the following conditions are satisfied.

- The expression contains no negative or zero exponents.
- The radicand contains no factor raised to a power equal to or greater than the index of the radical.
- The index of the radical is a small as possible.
- The denominator contains no radicals, and no fraction appears under a radical. (We will discuss this in a future section.)
Example 4: Simplify the following radical expressions.

a) \( \sqrt{27} \)

b) \( \sqrt[3]{16} \)

c) \( \sqrt[3]{40y^{10}} \)

d) \( \sqrt[12]{12r^9s^{12}} \)

e) \( \frac{7\sqrt[4]{162}}{\sqrt[4]{2}} \)

f) \( \sqrt[3]{54x^5} \)

g) \( \sqrt[4]{32x^6} \)