Exponential Functions

A function is called an __________________ if it has a constant growth/decay factor. For a fixed change in $x$, $y$ gets multiplied by a fixed amount. There are many applications that involve exponential functions, including population growth, temperature cooling, radioactive substances decay, virus spread, and compound interest.

**Exponential Functions**

A basic exponential function appears as: 

$$ f(x) = b^x \text{ where } x \text{ is a real number, } b > 0, b \neq 1 $$

To be an exponential function, the variable must be in the ______________. For example, $f(x) = x^2$ is not an exponential function!

We can graph the function $f(x) = 2^x$ using a table of values.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

On the graph of $f(x)$, notice that it never crosses the $x$ axis. It only approaches it. Why?
Look at the graphs of some other exponential functions.

\[ f(x) = 3^x \]
\[ g(x) = 4^x \]
\[ h(x) = 5^x \]

\[ F(x) = (1/3)^x \]
\[ g(x) = (1/4)^x \]
\[ h(x) = (1/5)^x \]

\[ \text{with } b > 1 \]
\[ \text{with } 0 < b < 1 \]

Functions of the form \( f(x) = b^x \) have the following characteristics:

Domain:

Range:

\( y \) intercept:

\( x \) intercept:

One-to-one (so their inverse is a function)

They all go through the point ________________.

\[ f(x) = 3^{x-1} \]

\[ \text{Domain:} \]

\[ \text{Range:} \]

\( y \) intercept:

\( x \) intercept:
$e$ is an irrational number (a decimal that never ends or repeats) and occurs in many math and science applications.

$$e \approx 2.7182818284590….\ldots$$

http://antwrp.gsfc.nasa.gov/htmltest/gifcity/e.2mil

There is an “$e$” key on your calculator that you can use for approximate calculations. The exponential function with base $e$, $y = e^x$, is very common in real life applications.

**Application: Compound Interest**

When you invest money into an account that pays compound interest, you earn interest on your interest as well as the principal. The formula is given below:

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

- $A$ = total end amount of money
- $P$ = principal (initial investment)
- $r$ = annual interest rate (as a decimal)
- $n$ = number of compounding periods per year
- $t$ = number of years

*Example 2*: How much money would you have in 10 years if you invested $100 right now in a savings account that pays 2.5% annual interest, compounded semi-annually?

**Continuous Compounding:**

We learned the formula for compound interest, where $n$ is the number of compounding periods per year. When money “compounds continuously”, it means that an exponential function with a base of $e$ is being used and the formula becomes:

$$A = Pe^rt$$ where $r$ is the annual interest rate and $t$ is the number of years.

*Example 3*: Find how much money you would have in 10 years if you invest $100 into an account that pays 2.5% annual interest, compounded continuously.